Core 4 - January 2007

1) a) i) $x = 1 + 2t$ $\qquad$ $y = 1 - 4t^2$

$dx/dt = 2$ $\qquad$ $dy/dt = -8t$

ii) $dy/dx = dy/dt \times dt/dx = -8t \times 1/2 = -4t$

b) $t = 1$, $x = 1 + 2(1) = 3$

$y = 1 - 4(1)^2 = -3$

$dy/dt = -4(1) = -4$, therefore gradient normal $= 1/4$

$y - y_1 = m(x - x_1)$

$y + 3 = 1/4(x - 3)$

$4y + 12 = x - 3$

$x - 4y - 15 = 0$

c) $x = 1 + 2t \rightarrow x - 1 = 2t \rightarrow t = \left(\frac{x-1}{2}\right)$

$y = 1 - 4t^2$

$y = 1 - 4\left(\frac{x-1}{2}\right)^2$

2) a) $2x - 3 \rightarrow$ sub in $x = 3/2$

$f(3/2) = 2(3/2)^3 - 7(3/2)^2 + 13 = 4$

b) $f(3/2) = 0$ if $2x - 3$ is a factor

$f(3/2) = 2(3/2)^3 - 7(3/2)^2 + 13 + d = 0$

$4 + d = 0 \rightarrow d = -4$

c) $(2x-3)(x^2 - 2x - 3) = 2x^3 - 7x^2 + 9$

$$
\begin{array}{r|llll}
 & x^2 & -2x & -3 & \\
\hline
(2x-3) & 2x^3 & -7x^2 & +0x & +9 \\
 & 2x^3 & -3x^2 & & \\
 & & -4x^2 & +6x & \\
 & & & -6x & +9
\end{array}
$$

$a = -2$
$b = -3$

3) a) $\cos(2x) = 1 - 2\sin^2(x)$

b) i) $3\sin(x) - \cos(2x)$

$= 3\sin(x) - (1 - 2\sin^2(x))$

$= 3\sin(x) - 1 + 2\sin^2(x)$

$= 2\sin^2(x) + 3\sin(x) - 1$

Be careful!!!

$3\sin(x) - \cos(2x) = 1$

$= 2\sin^2(x) + 3\sin(x) - 1 = 1$

$= 2\sin^2(x) + 3\sin(x) - 2 = 0$

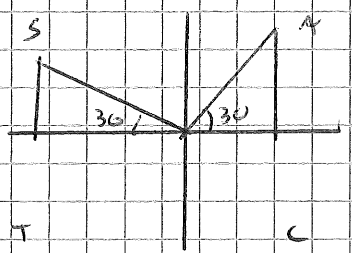
ii) $(2\sin(x) - 1)(\sin(x) + 2) = 0$

$\sin(x) = 1/2$

$x = 30°$

$\sin(x) = -2$

$x =$ NO REAL SOLUTIONS



$x = 30°, 150°$

c) $\int \sin^2(x)\, dx$

$\cos(2x) = 1 - 2\sin^2(x)$

$2\sin^2(x) = 1 - \cos(2x)$

$\sin^2(x) = 1/2\,(1 - \cos(2x))$

$= 1/2 \int 1 - \cos(2x)\, dx$

$= 1/2\,[x - 1/2\sin(2x)]$

$= x/2 - 1/4\sin(2x)$

4) a) i) $\dfrac{3x-5}{x-3} = A + \dfrac{B}{x-3}$

$3x - 5 = A(x-3) + B$

$\boxed{x=3}$ $4 = B$

$\boxed{x=0}$ $-5 = -3A + 4$

$3A = 9 \rightarrow A = 3$

$= 3 + \dfrac{4}{x-3}$

ii) $\int 3 + \dfrac{4}{x-3} = 3x + 4\ln(x-3) + C$

b) i) $$\frac{6x-5}{(2x+5)(2x-5)} = \frac{P}{(2x+5)} + \frac{Q}{(2x-5)}$$

$$6x-5 = P(2x-5) + Q(2x+5)$$

$\boxed{x = 5/2}$ $10 = 10Q \rightarrow Q = 1$

$\boxed{x = -5/2}$ $-20 = -10P \rightarrow P = 2$

$$\rightarrow \frac{2}{2x+5} + \frac{1}{2x-5}$$

ii) $$\int \frac{2}{2x+5} + \int \frac{1}{2x-5}$$

$$= \ln(2x+5) + \tfrac{1}{2}\ln(2x-5) + c$$

5) a) $$(1+x)^{1/3} \approx 1 + (1/3)(x) + \frac{(1/3)(-2/3)x^2}{2}$$

$$= 1 + x/3 - x^2/9$$

b) i) $$(8+3x)^{1/3} = 8^{1/3}(1 + 3/8\,x)^{1/3}$$

$$= 2\left[1 + (1/3)(3/8\,x) - 1/9\,(3/8\,x)^2\right]$$

$$= 2\left[1 + 1/8\,x - 1/64\,x^2\right]$$

$$= 2 + 1/4\,x - 1/32\,x^2$$

ii) $(8+3x)^{1/3}$ if $x = 1/3 \rightarrow 9^{1/3}$

$$\approx 2 + 1/4\,(1/3) - 1/32\,(1/3)^2$$

$$= \frac{599}{288}$$

6) c) i) $$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$$

$$= \begin{pmatrix} -5 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \\ 4 \end{pmatrix}$$

ii) $$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = \begin{pmatrix} -5 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 11 \\ 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix}$$

$$|\overrightarrow{BA}| = \sqrt{(-2)^2 + (-6)^2 + (4)^2} = \sqrt{56}$$

$$|\overrightarrow{BC}| = \sqrt{56}$$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = \begin{pmatrix} -2 \\ -6 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix} = -12 - 12 - 16 = -40$

$\cos(ABC) = \dfrac{a \cdot b}{|a||b|} = \dfrac{-40}{\sqrt{56}\sqrt{56}} = \dfrac{-40}{56} = -5/7$

$\therefore ABC = \cos^{-1}(-5/7)$

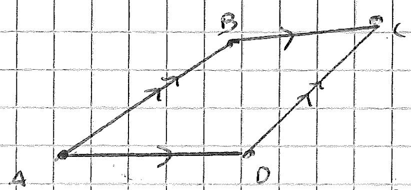
b) $r = \begin{pmatrix} 8 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

i) $c = \begin{pmatrix} 11 \\ 6 \\ -4 \end{pmatrix}$ $\quad 8 + 1\lambda = 11$, $\quad -3 + 3\lambda = 6$, $\quad 2 - 2\lambda = -4$ All satisfied by $\lambda = 3$

$\therefore$ C lies on line l

ii) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ $\therefore$ parallel to l as same direction

c)



$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$

$= \begin{pmatrix} 11 \\ 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -2 \\ -6 \\ 4 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \\ 0 \end{pmatrix}$

$D = (9, 0, 0)$

7) a) $A = B = x \rightarrow \tan(2x) = \dfrac{\tan(x) + \tan(x)}{1 - \tan(x)\tan(x)}$

$= \dfrac{2\tan(x)}{1 - \tan^2(x)}$

b) $2 - 2\tan(x) - \dfrac{2\tan(x)}{\tan(2x)}$

$= 2 - 2\tan(x) - 2\tan(x)\dfrac{(1 - \tan^2(x))}{2\tan(x)}$

$= 2 - 2\tan(x) - (1 - \tan^2 x)$

$= 2 - 2\tan(x) - 1 + \tan^2(x)$

$= \tan^2(x) - 2\tan(x) + 1$

$= (\tan(x) - 1)(\tan(x) - 1)$

$= (\tan(x) - 1)^2$

(8) a) i) $dy/dt = y\sin(t)$

$$\int 1/y \, dy = \int \sin(t) \, dt$$

$$\ln(y) = -\cos(t) + c$$

$$y = e^{-\cos(t)+c}$$

$$y = Ae^{-\cos(t)} \qquad \text{where } A = e^c$$

ii) $y = 50,\ t = \pi \rightarrow 50 = Ae^{-\cos(\pi)}$

$$50 = Ae^1 \rightarrow A = 50/e$$

$$y = 50/e \ e^{-\cos(t)}$$

$$= 50e^{-1} e^{-\cos(t)}$$

$$= 50 e^{(-1 + -\cos(t))}$$

$$= 50 e^{-(1+\cos(t))}$$

b) i) $y = 50e^{-(1+\cos(t))}$

$$t = 6 \rightarrow y = 50e^{-(1+\cos(6))}$$

$$= 7.0417\ldots = 7.04 \text{ cm (2dp)}$$

$$= 7 \text{ cm (nearest cm)}$$

ii) $d^2y/dt^2 = \cancel{}\,dy/dt \sin(t) + y\cos(t)$

$u = y$	$v = \sin(t)$
$du/dy = dy/dt$	$dv/dt = \cos(t)$

PRODUCT RULE

$$d^2y/dt^2 = dy/dt \sin(t) + y\cos(t)$$

when $t = \pi$ $\quad dy/dt = \overset{50}{y} \times \overset{0}{\sin(\pi)} = 0$

$\therefore$ stationary point

when $t = \pi$ $\quad d^2y/dt^2 = 0 + 50 \times \cos(\pi)$

$$= -50$$

$\therefore$ MAXIMUM